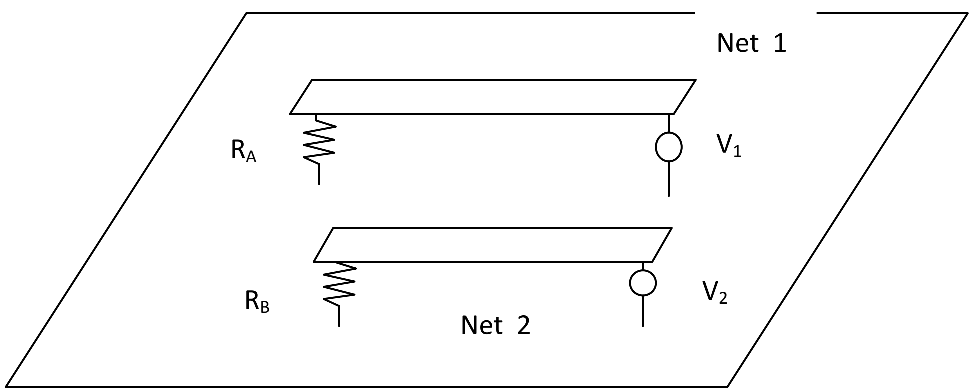
The Interpretation of Non-Zero Mutual Resistances in nApex

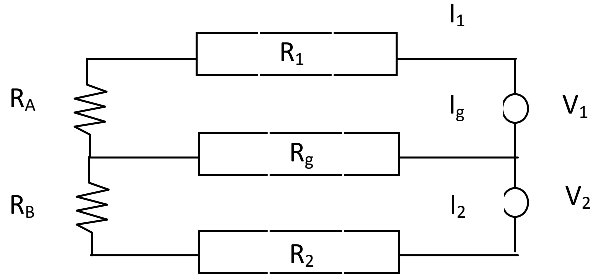
by Vikram Jandhyala and Swagato Chakraborty

Nimbic's accelerated system- in-package extractor nApex rapidly creates accurate and large-scale full-net RLGC models. This note discusses the physical meaning of the mutual resistance terms generated. It is shown that these terms have explicit correspondence to overlapping ground returns at low frequency. The concept of high-frequency induced-current-based mutual resistance is also shown to be erroneous. Finally, the distinction between transmission-line like nets, with a common ground, and low-frequency conductor nets, is shown to be the reason for the appearance of the mutual resistance terms.

Consider two nets that *share* the same conductor. This might happen if the two nets have a common ground reference which is modeled explicitly. Explicit ground models are possible in nApex and can be used because of the speed and scalability of the solver. A simple example is shown below.



Clearly, the two nets above share a common conductor. The ground return paths may have an overlap. In fact, in this case, the overlap will be larger at lower frequencies where the return currents tend to spread out and not follow a tight path under the trace. In the low frequency (quasi-static) case, we can write the equivalent circuit (where the common part of the ground return is shown as the middle segment. Assume we are looking only at the resistive part of the problem.



The Interpretation of Non-Zero Mutual Resistances in nApex

The voltages are with respect to the central conductor (representing the shared ground), and the currents are assumed to be directed away from the sources towards the loads on the left.

Consider the conductance representation of the two net case (the inverse of this 2X2 matrix will provide the resistance matrix.

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

We use smaller case letters in the matrix so as not to confuse with the conductance matrix associated with dielectric losses. To find the off-diagonal term in the first row, we will short the second port while exciting the first with a unit positive voltage, i.e.

$$g_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

We now show that this term will in general be non-zero. From KCL, first we see that

$$I_g = I_1 + I_2$$

Now we apply KVL around the two nets to get:

$$V_1 - (R_A + R_1)I_1 + R_G I_g$$

And

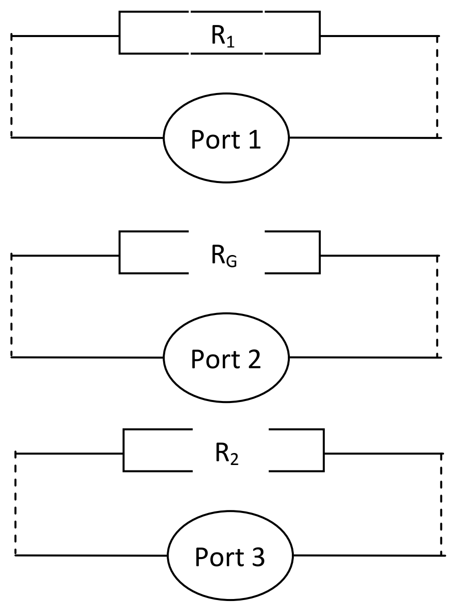
$$0 - (R_B + R_2)I_2 + R_G I_g$$

From the definition of g_{21}, the only way it will be zero is if I_2 is zero. From the equation above, that implies I_g must be zero, and therefore I_1 is zero. Therefore in the non-trivial case of non-zero current flow through net1, the cross coupling term *must* be non-zero. In exactly the same manner, it can be shown that g_{12}must be non-zero.

As is seen, the only way that the off-diagonal term is zero is when there is no current through the ground return. This only happens in the trivial case when there is no current flow anywhere in the circuit. The non-zero off-diagonal terms in the inverse of the resistance matrix hence lead to non-zero mutual terms in the resistance matrix.

The physical reason for this non-zero resistance, even at low frequencies (and DC) is the shared ground return between the two nets. These two nets have been setup in a transmission-line like topology, where there is a shared reference conductor. In general, in an N+1 multiconductor line system, if the 0^{th} line is chosen as a reference, the resistance matrix is an NXN matrix, where *all* entries in the matrix (including diagonal as well) are incremented by the value of the resistance in the reference conductor path. This is exactly the same effect happening in the 3D case described here.

An alternative definition of ports, only valid at quasi-static (and static) limits, would be to treat every conductor separately, and define ports *across* the conductors. Of course this port definition is invalid (electrically large!) at high frequencies. In that case, we would arrive at a 3X3 resistance matrix with only diagonal values. This is shown below. Consider three conductors with ports defined across the conductors.



This configuration will not have any off-diagonal resistances, and the resistance matrix will be

$$\begin{pmatrix} R_1 & 0 & 0 \\ 0 & R_G & 0 \\ 0 & 0 & R_2 \end{pmatrix}$$

However, once there is a shared return conductor, the resistance of the return conductor effects every element of the resistance matrix, including the off-diagonal elements, creating mutual resistances.

Summary: Mutual resistances are created by shared conductors (typically ground returns) in port/net definitions. These are typically avoided in some formulations where ports are defined across each conductor (net). However, those definitions come at the cost of only being valid at static or quasi-static limits.

The Interpretation of Non-Zero Mutual Resistances in nApex

Passivity

A question may arise as to whether the cross-resistance can have arbitrary sign, and what this does to passivity. Depending on the current flow, it is true that the cross-resistance may be positive or negative (much like mutual inductance, but unlike mutual capacitance). However, it is easy to show that passivity is not violated by the sign change.

The cross term contribution to power created by the off-diagonal resistance term R_{ij} is equal to the product of the induced voltage in net *I* due to the current in net *j* times the current in net *I*:

$$I_i R_{ij} I_j$$

If this is positive , passivity is preserved. Assume we start with a case where all 3 terms have the same positive sign. Now let us change the current direction convention in either net. As can be seen by the definition of the off-diagonal admittance/resistance terms defined earlier, this will have the effect of changing the sign of R_{ij}if either current changes sign, but not if both change sign. Therefore the product of the 3 terms will continue to remain positive, ensuring passivity.

> *Summary*: Mutual resistances can have arbitrary sign. However, the signs of the resistances, and the signs of the currents flowing through the two associated nets are linked such that the power loss associated with the mutual resistance is always positive and passivity is maintained.

The fallacy of an induced resistive coupling term at high (or low) frequencies

Confusion often arises in EM modeling circles and in some competing products as to the definition of off-diagonal resistances at high frequency. The argument runs as follows: An excited port on net 1 generates a field on net 2 (especially at high frequency). Since net 2 is lossy, the induced field generates a current which experiences travel through a lossy net. This is the justification for the resistive cross coupling. The simple fallacy is that the cross-coupling is actually through the induced field, not the loss! It is the mutual inductive (or capacitive) coupling that generates the induced field. The resulting induced current then experiences loss through the *self resistance* of net 2.